Enrollment No:-__________ Exam Seat No:-_______

C.U.SHAH UNIVERSITY

Summer-2015

Subject Code: **4SC03MTC2** Subject Name: Linear Algebra-I

Course Name:B.Sc(Pure Science) Date :5/5/2015

Semester:3 Mark: 70

Time:2:30To 5:30

Instructions:

1) Attempt all Questions of both sections in same answer book/Supplementary.
2) Use of Programmable calculator & any other electronic instrument prohibited.
3) Instructions written on main answer book are strictly to be obeyed.
4) Draw neat diagrams & figures (if necessary) at right places.
5) Assume suitable & perfect data if needed.

SECTION-I

Q-1 a) Define: Vector space. (02)

b) Let $T: R^n \to R$ be a map defined by $T(x_1, x_2, \dots, x_n) = x_1$, prove that T is a linear Transformation. (02)

c) Show that $\{(1,1,1), (1,1,0), (1,0,0)\}$ is linearly independent in R^3. (02)

d) Show that the vectors $A_1 = \begin{bmatrix} -3 & 4 \\ 2 & 0 \end{bmatrix}$ and $A_2 = \begin{bmatrix} 3 & -4 \\ -2 & 0 \end{bmatrix}$ is linearly dependent. (01)

Q-2 a) Show that R^n is vector space with the standard operations. (07)

b) Obtain the matrix of a linear transformation $T: R^3 \to R^3$ defined by $T(a,b,c) = (2a, a+2b+c, a+3c)$ with respect to the basis $B_1 = \{(1,0,0), (1,1,0), (1,1,1)\}$and $B_2 = \{(1,0,0), (0,1,0), (0,0,1)\}$. (04)

c) Show that $W = \{(x_1, x_2): x_1 = 3x_2\}$ is a subspace ofR^2. (03)

OR

Q-2 a) Show that$F[a,b]$, the set of all real valued functions on $[a,b]$ is a vector space with the standard operations. (07)

b) Obtain the matrix of a linear transformation$T: p_3 \to p_2$ be defined by $T(a_0 + a_1x + a_2x^2 + a_3x^3) = a_3 + (a_2 + a_3)x + (a_0 + a_1)x^2$ with respect to the basis$B_1 = \{1, (x-1), (x-1)^2, (x-1)^3\}$ and $B_2 = \{1, x, x^2\}$. (04)

c) Show that $S = \{a_0 + a_1x + a_2x^2 + a_3x^3 | a_0, a_1, a_2, a_3 \in R,$ (03)

$$a_0 + a_1 + a_2 + a_3 = 0\}$$

is a subspace of p_3, the space of all polynomials of degree 3 or less.

Q-3 a) Determine whether the following maps are linear transformations (05)

$i)\ T: M_{mn} \to M_{nm}$, where $T(A) = A'$

$ii)\ T: M_{22} \to R$, where $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a^2 + b^2$

b) Consider the basis $S = \{v_1, v_2\}$ for R^2, where $v_1 = (1,1)$ and $v_2 = (1,0)$ and let $T: R^2 \to R^2$ be the linear transformation such that $T(v_1) = (1,-2)$ and $T(v_2) = (-4,1)$. Find a formula for $T(x_1, x_2)$ and use this formula to find $T(5,-3)$. (05)

	c)	Determine whether $v_1 = (2,2,2), v_2 = (0,0,3)$ and $v_3 = (0,1,1)$ span the vector space R^3.	(04)

OR

Q-3	a)	Show that the set $S = \{v_1, v_2, v_3, v_4\}$ is a basis for the vector space M_{22} where, $v_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, v_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, v_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$	(05)
	b)	Let V and W be a vector spaces. Prove that the dimension of $L(V, W)$ is $\dim V \times \dim W$.	(05)
	c)	The map $T: R^3 \to R^2$ defined by $T(x, y, z) = (2x - y + z, y - 4z)$, prove that $T$is a linear transformation.	(04)

SECTION-II

Q-4	a)	Let $u = (1,2,3), v = (-2,1,2)$, find$< 2u + 3v, u + v >$.	(02)
	b)	Define: Kernel and Range.	(02)
	c)	If u and v are non-zero vectors in an inner product space and θ is the angle between u and v , $< u, v > = 0$ iff $\theta = \frac{\pi}{2}$.	(02)
	d)	Write standard matrix of identity operator.	(01)
Q-5	a)	For linear transformation $T: R^3 \to R^3, T(x, y, z) = (x - y + z, y - z,\ z)$ for all (x, y, z). Find T^{-1} if it exists.	(05)
	b)	Let V and W be finite dimensional vector spaces and $T: V \to W$ be a linear transformation.Prove that $\dim V = rank(T) + nullity(T)$.	(05)
	c)	Let $T: V \to W$ be the linear transformation. If $V = sp\{v_1, v_2, \dots, v_n\}$, prove that $Im(T) = sp\{T(v_1), T(v_2), \dots, T(v_n)\}$.	(04)

OR

Q-5	a)	Let V be any vector space over R. Let $T: V \to V$ be defined by $T(v) = kv, k \neq 0$, prove that T is a linear isomorphism.	(05)
	b)	If V and W are finite-dimensional vector spaces of the same dimension, prove that a linear map $T: V \to W$ is one-one if and only if it is onto.	(05)
	c)	Show that composition of two linear transformation is a linear transformation.	(04)
Q-6	a)	Let $u = (3, -1), v = (2, -2), w = (-1, 6)$ and $k = -2$. Verify the following using Euclidean inner product. i) $< u + v, w > = < u, w > + < v, w >$ ii) $< ku, v > = k < u, v >$ iii) $< 0, u > = < u, 0 > = 0$	(05)
	b)	Let $T: R^3 \to R^3$ be the linear transformation defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$.Find $\ker(T)$, $Im(T), n(T)$, $r(T)$.	(05)
	c)	Let P_2 be the vector space with inner product. $< p, q > = \int_{-1}^{1} p(x)\, q(x)\, dx$. If $p(x) = 1$and $q(x) = x$. Show that p and q are orthogonal and verify the Pythagorean Theorem.	(04)

OR

Q-6	a)	Determine whether there exists scalars k and l such that the vectors $u = (2, k, 6), v = (l, 5, 3)$ and $w = (1,2,3)$ are mutually orthogonal with respect to the Euclidean inner product.	(05)
	b)	Let V be an inner product space, $\|u\| = < u, u >^{\frac{1}{2}}$ satisfies the following	(05)

properties for all $u, v \in V$ and for any scalar k.

i) $\|u\| \geq 0$ and $\|u\| = 0$ iff $u = 0$

ii) $\|ku\| = |k|\|u\|$

iii) $\|u + v\| \leq \|u\| + \|v\|$

c) Let u, v, w be a vectors in a real inner product space V and k is any scalar. Then show that (04)

i) $< u, v + w > = < u, v > + < u, w >$

ii) $< u, kv > = k < u, v >$